ECE 330 Exam 2: Summer 2019 NAME______________________________
90 minutes J. Schuh

1. __________/25 2. __________/25

3. __________/25 4. __________/25 TOTAL _________/100

USEFUL INFORMATION

sin(x)=cos(x-90°) $\bar{V} = \bar{Z}\bar{I}$ $\bar{S} = \bar{V}\bar{I}^* = P + jQ$ $\bar{S}_{3\varphi} = \sqrt{3}V_L I_L \angle\theta$

$0<\theta<180°$ (lag) $I_L = \sqrt{3}I_\varphi$ (delta) $\bar{Z}_Y = \bar{Z}_\Delta/3$

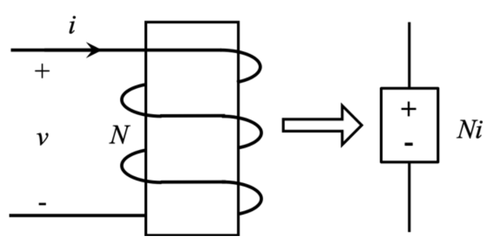
$-180°<\theta<0$ (lead) $V_L = \sqrt{3}V_\varphi$ (wye) $\mu_0 = 4\pi \times 10^{-7}$ H/m

ABC phase sequence has A at 0, B at -120°, and C at +120°

$\int \underline{H} \cdot \underline{dl} = \int \underline{J_f} \cdot \hat{n} dA$ $\int \underline{E} \cdot \underline{dl} = -\frac{d}{dt}\left(\int \underline{B} \cdot \hat{n} dA\right)$ $\mathcal{R} = \frac{l}{\mu A}$ $Ni = \mathcal{R}\varphi$

$\varphi = BA$ $\lambda = N\varphi = Li$ (if linear) $v = \frac{d\lambda}{dt}$ $k = \frac{M}{\sqrt{L_1 L_2}}$

1hp=746 W



$W_m = \int_0^\lambda i d\hat{\lambda}$ $W_m' = \int_0^i \lambda d\hat{\imath}$ $W_m + W_m' = i\lambda$ $f^e = -\frac{\partial W_m}{\partial x} = \frac{\partial W_m'}{\partial x}$

$x \to \theta, f^e \to T^e$

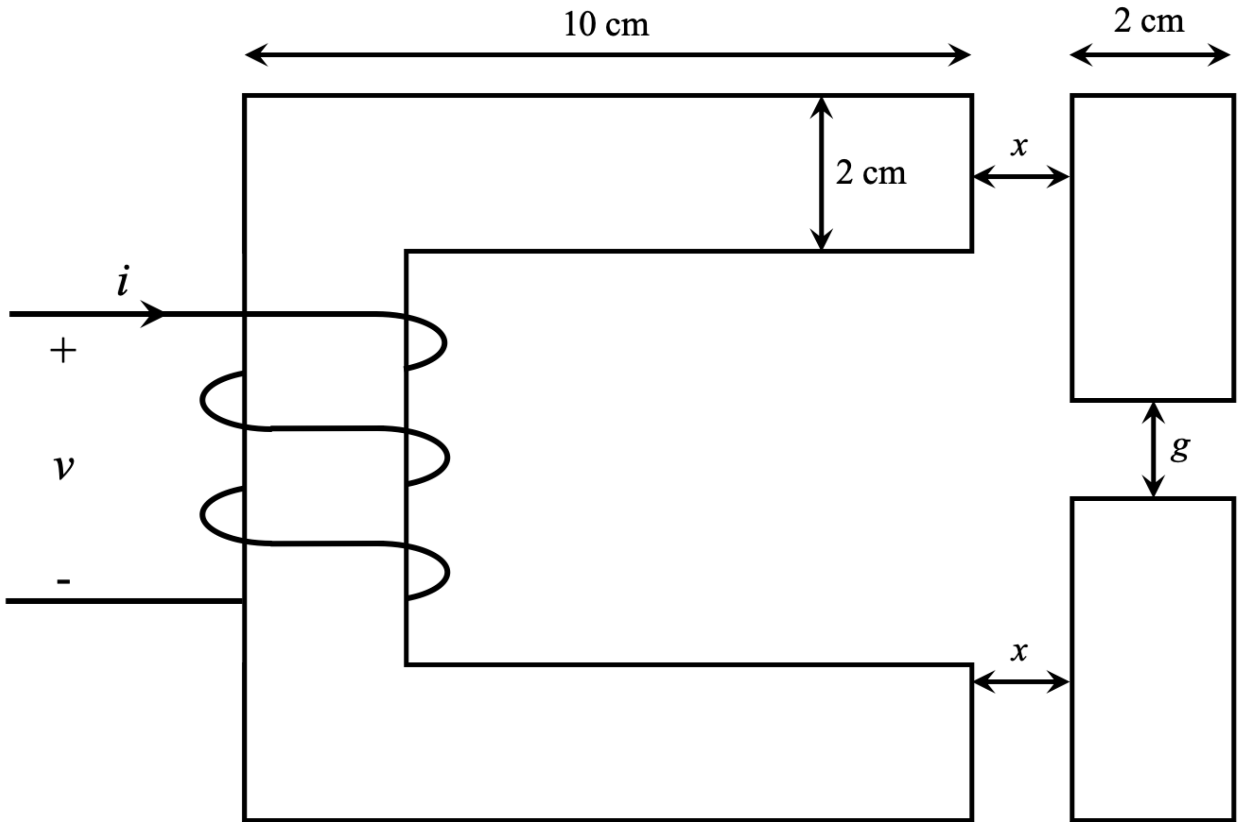
$EFE_{a\to b} = \int_a^b i d\lambda$ $EFM_{a\to b} = \int_a^b -f^e dx$ $EFE_{a\to b} + EFM_{a\to b} = W_{mb} - W_{ma}$

$i = \frac{\partial W_m}{\partial \lambda}$ $\lambda = \frac{\partial W_m'}{\partial i}$

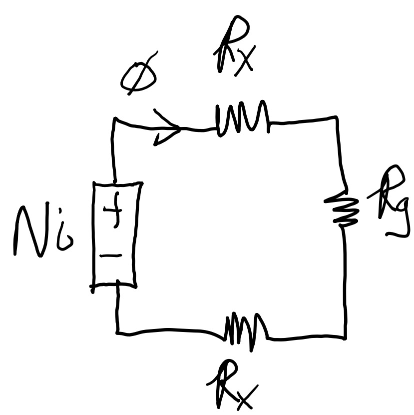
$\dot{x}_1 = f_1(x_1, x_2)$ $\dot{x}_2 = f_2(x_1, x_2)$ $x(t_0 + \Delta t) \approx x(t_0) + \Delta t \left.\frac{dx}{dt}\right|_{t=t_0}$

Problem 1 (25 Points)

The system below has a coil wound 250 times around an iron core with infinite permeability (dimensions given) that has an air gap with a constant length of 5 mm and a depth into the page of 3 cm. Neglect fringing. The piece with the coil is fixed while the other pieces can move.



a) What is the flux linkage of the coil? (8 points)



$$Ni = (2R_x + R_g)\phi$$

$$\phi = \frac{Ni}{2R_x + R_g} \qquad \lambda = \frac{N^2 i}{2R_x + R_g}$$

$$R_x = \frac{x}{\mu_0 (6\,cm^2)} \Rightarrow R_x = 1.326\times10^9\,x \ \ At/Wb$$

$$R_g = \frac{g}{\mu_0 (6\,cm^2)} \Rightarrow R_g = 1.326\times10^9\,g \ \ At/Wb$$

$$\lambda = \frac{250^2\, i}{2(1.326\times10^9 x) + 1.326\times10^9 g} \Rightarrow \boxed{\lambda = \frac{0.00004713}{2x+g}\, i}$$

b) What is the expression for the voltage across the coil in terms of *x* and *i*? (8 points)

$$V = \frac{d\lambda}{dt} \Rightarrow V = \frac{0.00004713}{2x+g}\frac{di}{dt} - \frac{0.00004713}{(2x+g)^2}(2)i\frac{dx}{dt}$$

$$\boxed{V = \frac{0.00004713}{2x+g}\frac{di}{dt} - \frac{0.00009426}{(2x+g)^2}\frac{dx}{dt}}$$

c) Find W_m and f^e in terms of *x* and λ (9 points)

$$i = \frac{(2x+g)}{0.00004713}\lambda$$

$$i = 21216(2x+g)\lambda$$

$$W_m = \int_0^{\lambda} i\,d\hat{\lambda}$$

$$W_m = \frac{1}{2}(21216)(2x+g)\lambda^2$$

$$\boxed{W_m = 10608(2x+g)\lambda^2}$$

$$f^e = -\frac{\partial W_m}{\partial x}$$

$$f^e = -10608(2)\lambda^2$$

$$\boxed{f^e = -21216\lambda^2}$$

<u>Problem 2 (25 Points)</u>

A rotor stator system has flux linkage given as

$$\lambda_s = L_s i_s + M\left[\cos(\theta) + \frac{1}{9}\cos(3\theta) + \frac{1}{25}\cos(5\theta)\right] i_r$$

$$\lambda_r = M\left[\cos(\theta) + \frac{1}{9}\cos(3\theta) + \frac{1}{25}\cos(5\theta)\right] i_s + L_r i_r$$

a) What is the co-energy of the rotor stator system? (8 points)

$$W_m' = \int_0^{i_s} \lambda_s(\hat{i}_s, i_r = 0, \theta)\, d\hat{i}_s + \int_0^{i_r} \lambda_r(i_s, \hat{i}_r, \theta)\, d\hat{i}_r$$

$$W_m' = \frac{1}{2} L_s i_s^2 + \frac{1}{2} L_r i_r^2 + M\left[\cos(\theta) + \frac{1}{9}\cos(3\theta) + \frac{1}{25}\cos(5\theta)\right] i_s i_r$$

b) What is the torque of electric origin? (9 points)

$$T^e = \frac{\partial W_m'}{\partial \theta}$$

$$T^e = -M\left[\sin(\theta) + \frac{1}{3}\sin(3\theta) + \frac{1}{5}\sin(5\theta)\right] i_s i_r$$

c) What is the torque of electric origin if i_s=10 A, i_r=1 A, M=1 H, and

i) θ=0° (3 points)

$T^e = 0$

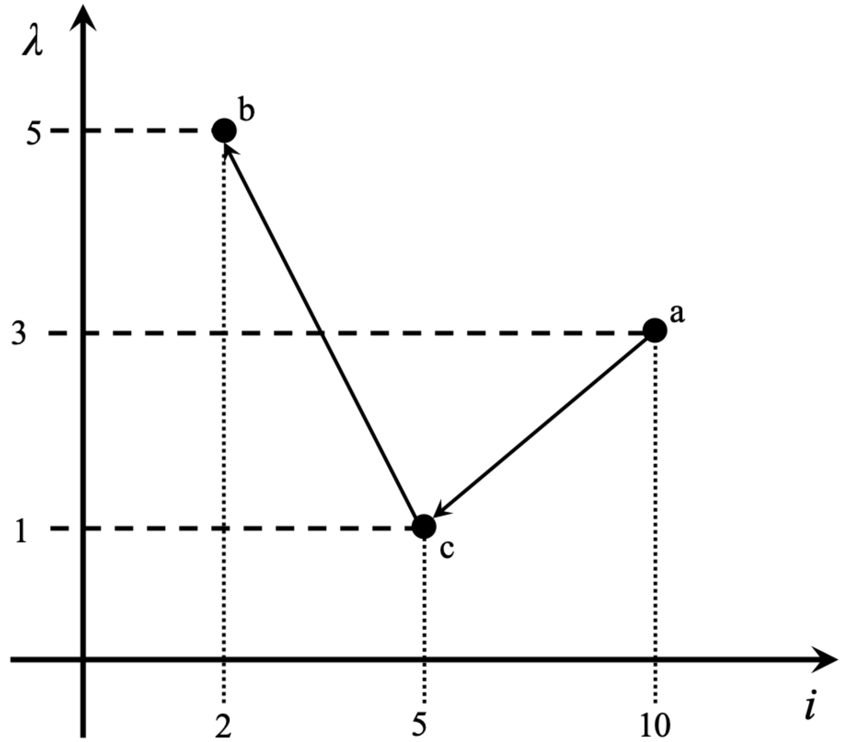
iii) θ=180° (3 points)

$T^e = 0$

d) What is problematic about the torque produced in this system? (2 points)

It depends on the angle θ.

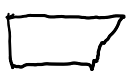
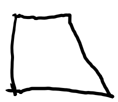
Problem 3 (25 Points)



An electromechanical system with $\lambda = L(x)i$ is moved from point a to b through point c as shown. The current is given as Amps and the flux linkage is given as Amp turns.

a) Find the energy transferred from the electrical system to the coupling field as the system moves from a to b. (8 points)

$$EFE_{a \to b} = \int_{\lambda_a}^{\lambda_b} i\,d\lambda = \int_{\lambda_a}^{\lambda_c} i\,d\lambda + \int_{\lambda_c}^{\lambda_b} i\,d\lambda$$

= −  + 

$$= -\frac{(3-1)}{2}(5+10) + \frac{(5-1)}{2}(2+5)$$

$$= -1(15) + 2(7)$$

$$= -15 + 14$$

$$\boxed{EFE_{a \to b} = -1\,J}$$

b) Find the energy transferred from the mechanical system to the coupling field as the system moves from a to b. (9 points)

$$W_{mb} - W_{ma} = \underset{a\to b}{EFE} + \underset{a\to b}{EFM}$$

$$\underset{a\to b}{EFM} = W_{mb} - W_{ma} - \underset{a\to b}{EFE}$$

$$W_{mb} = \frac{1}{2}(5)(2) = 5J$$

$$W_{ma} = \frac{1}{2}(3)(10) = 15J$$

$$\underset{a\to b}{EFM} = 5 - 15 - (-1) \Rightarrow \boxed{\underset{a\to b}{EFM} = -9J}$$

c) If the system moves back from b to a along a straight line, is the system acting as a generator or a motor? (8 points)

$$\underset{cycle}{EFE} = \underset{a\to b}{EFE} + \underset{b\to a}{EFE}$$

$$\underset{b\to a}{EFE} = -\frac{(5-3)}{2}(2+10)$$

$$= -1(12)$$

$$= -12J$$

$$\underset{cycle}{EFE} = -13J$$

generator

$$\underset{cycle}{EFM} = 13J$$

Problem 4 (25 Points)

The equations of motion for a pendulum attached to a cart are given as

$$\ddot{x} = \frac{-x + \frac{1}{2}\left(\dot{\theta}^2 \sin(\theta) + \sin(\theta)\cos(\theta)\right)}{1 + \frac{1}{2}\sin^2(\theta)}$$

$$\ddot{\theta} = -\sin(\theta) + \frac{x - \frac{1}{2}\left(\dot{\theta}^2 \sin(\theta) + \sin(\theta)\cos(\theta)\right)}{1 + \frac{1}{2}\sin^2(\theta)}$$

where x is the position of the cart and θ is the angle of the pendulum.

a) Write the equations of motion in state space form. (8 points)

$x_1 = x$

$x_2 = \dot{x}$

$x_3 = \theta$

$x_4 = \dot{\theta}$

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = \frac{-x_1 + \frac{1}{2}\left(x_4^2 \sin(x_3) + \sin(x_3)\cos(x_3)\right)}{1 + \frac{1}{2}\sin^2(x_3)}$$

$$\frac{dx_3}{dt} = x_4$$

$$\frac{dx_4}{dt} = -\sin(x_3) + \frac{x_1 - \frac{1}{2}\left(x_4^2 \sin(x_3) + \sin(x_3)\cos(x_3)\right)}{1 + \frac{1}{2}\sin^2(x_3)}$$

b) Find the equilibrium positions of the physical system. (8 points)

$0 = x_2$

$$0 = \frac{-x_1 + \frac{1}{2}\left(x_4^2 \sin(x_3) + \sin(x_3)\cos(x_3)\right)}{1 + \frac{1}{2}\sin^2(x_3)}$$

$0 = x_4$

$$0 = -\sin(x_3) + \frac{x_1 - \frac{1}{2}\left(x_4^2 \sin(x_3) + \sin(x_3)\cos(x_3)\right)}{1 + \frac{1}{2}\sin^2(x_3)}$$

$\Rightarrow 0 = x_2$

$0 = x_4$

$0 = -\sin(x_3)$

$0 = -x_1$

Equilibrium points

$x_1 = 0$

$x_2 = 0$

$x_3 = n\pi$

$x_4 = 0$

n is an integer

c) Using a time step of 0.005 s, find the position of the cart and the angle of the pendulum at t=0.01 s. Use the initial conditions: $x(t=0)=0$, $\dot{x}(t=0)=1$ m/s, $q(t=0)=0$ rad, and $\dot{\theta}(t=0)=1$ rad/s. (9 points)

$$x_1^n = x_1^{n-1} + \Delta t\, x_2^{n-1}$$

$$x_2^n = x_2^{n-1} + \Delta t\left(\frac{-x_1^{n-1} + \frac{1}{2}\left(\left(x_4^{n-1}\right)^2 \sin\left(x_3^{n-1}\right) + \sin\left(x_3^{n-1}\right)\cos\left(x_3^{n-1}\right)\right)}{1+\frac{1}{2}\sin^2\left(x_3^{n-1}\right)}\right)$$

$$x_3 = x_3^{n-1} + \Delta t\, x_4^{n-1}$$

$$x_4 = x_4^{n-1} + \Delta t\left(-\sin\left(x_3^{n-1}\right) + \frac{x_1^{n-1} - \frac{1}{2}\left(\left(x_4^{n-1}\right)^2 \sin\left(x_3^{n-1}\right) + \sin\left(x_3^{n-1}\right)\cos\left(x_3^{n-1}\right)\right)}{1+\frac{1}{2}\sin^2\left(x_3^{n-1}\right)}\right)$$

t	x_1	x_2	x_3	x_4
0	0	1	0	1
0.005	0.005	1	0.005	1
0.01	0.01	1	0.01	0.999975

(Blank page for extra work)